B.Sc. (Honours) Examination, 2019
Semester-V
Statistics
Course : CC-11
(Stochastic Process and Queuing Theory)

Time : 3 Hours **Full Marks : 40**

Questions are of value as indicated in the margin.

Answer **any four** of the following questions

1. a) Define strict stationarity for a stochastic process $\{y_t; t \in T\}$. Does strict stationarity imply covariance stationary? Cite an example in favour of your answer. 2+3=5

   b) Indianapolis, the capital of Indiana, is not blessed by good weather consistently. If it has a nice day, it is as likely to have snow as rain the next-day. If it has snow or rain, there is an ¼ th chance of having the same the next days. If there is change from snow or rain, only half of the time is this a change to a nice day. Also, there never are two nice days in rows. Calculate transition prob.matrix.

   What is the chance of having nice day on Wednesday if today (Monday) is snowy. 2+3=5

2. a) What is an ergodic state? 2

   b) For the transition matrix on the state space {S=1,2,3,4} answer the following.

$$\begin{pmatrix} \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

   i) Find transient states and persistent states.

   ii) What is the period of all persistent state?

   iii) What is the expected number of steps required to get back to state 1?

   iv) Is state 1 ergodic? 3+1+2+2=8

3. a) Suppose a gambler went to a gambling zone with 10 unit of money. He gained one unit of money if he wins otherwise he lost one unit. He quit when he ran out or he earned 20 unit of money. Construct the difference equation of winning. Hence find out the probability of being winner. 6

   b) State True or False for the following statements

   i)Two communicating states sometimes have same periodicity.

   ii) In an finite irreducible chain all states are positive non null.

   iii) If limiting distribution exists for a Markov chain stationary distribution also exists.

P.T.O.

iv) For a Poisson process the sequence of interarrival times is a sequence of only independent variables. 4

4. a) Define a pure birth process. Also derive the Yule-Furry birth model clearly stating the assumptions you made. 2+4=6

b) For the following transition matrix on state space S={1,2,3,4,5} find $\underset{n\to\infty}{lt}\ p_{23}^{(n)}$.

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ 4

5. For the M/M/1 queuing system find the expected number of customers in the system in steady state and also the expected queue length. Find the cumulative distribution function for the waiting time of a customer who has to wait in an M/M/1 queuing system. 5+5=10

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